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An abstract subject

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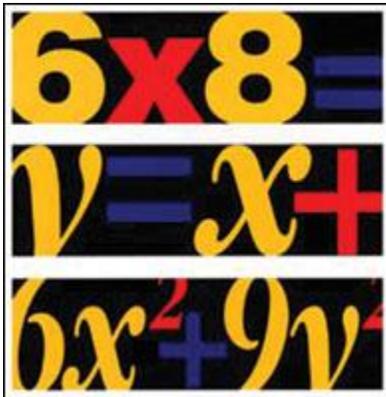
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An abstract subject
By Mashooque Ali Samo

Algebra is described as a gate keeper to the success of those interested in a career in mathematics and science. Making algebra accessible to all the students is a critical issue in mathematics education, not just for the high school mathematics teachers but also for elementary and middle-grade mathematics teachers. The National Council of Teachers of Mathematics (NCTM) believes that all students should learn algebra and that its study should be a part of the mathematics curriculum right from early education through grade 12.

Introducing algebraic concepts informally in the early grades should help students develop algebraic reasoning skills and begin to build a firm foundation for the more formal study of algebra in high school. But beside the significances of algebra, it is supposed to be a difficult area of mathematics. Students and teachers come across many difficulties in the teaching and learning of algebra, the use of symbols, letters and signs being one of them.

Algebra is a special language with its own conventions. Teachers need to recognise the fact that students do not come into an algebra classroom without any previous experience of using letters and other signs in different contexts. This prior exposure can be a factor of misunderstanding. Stacey and MacGregor (1997) have highlighted several causes for the common misunderstandings among students:

- Students' interpretations of algebraic symbolism are based on other experiences that are not helpful.

- The use of letters in algebra is not the same as their use in other contexts.

- The grammatical rules of algebra are not the same as ordinary language rules.

- Algebra cannot say a lot of the things that students want it to say.

Research studies have shown that the interpretation of algebraic expirations, particularly in the use of algebraic codes is not easy for many students. These studies have investigated that the majority of students were unable to interpret algebraic letters as generalised or even as specific unknown numbers.

The study of Kuchemann (1981) shows that many students ignore the letters, replace them by numerical values or regard them as shorthand of names or measurement labels. Arithmetic experiences in elementary schools may also lead one to different alternative frameworks in algebra. For instance, in arithmetic, the letters denote measurements. For example, 10m denotes 10 metres, but in algebra it may denote 10 times of an unspecified number. Traditionally children have limited experience with letters in elementary schools to equations such as a = lxw which shows the use of letters as labels in arithmetic. Using letters as measurement labels lead children to alternative frameworks to treat numerical variables as if they stood for the objects rather than the numbers.

A study by the Concept of Secondary Mathematics and Science (CSMS) project investigated the performance of school students aged 11-16 years on test items concerning the use of algebraic letters in generalised arithmetic. The results show that most of the students were unable to cope with items which require interpreting the letters as general numbers or specific unknowns. He also found the interpretation issue of pertaining letters in algebra. The study highlighted that students' misunderstanding of the letters seem to be reflected in their approach to the relevant relationship in problem solution.

The CSMS project reported that students were asked the question:

"Blue pencils cost five pence each and red pencils cost six pence each. I buy some blue and some red pencils and altogether it costs me 90 pence. If b is the number of blue pencils bought, and r is the number of red pencils bought, what can you write down about b and r?"

The most common response was b + r = 90. This mistake suggests a strong tendency to conceive letters as labels denoting specific sets, which seems to be a result of the students' attempt to accommodate their previous arithmetic experience with letters to the new meanings assigned to letters within an algebraic context.

Similar results were found in the National Assessment of Educational Progress, which was carried out with 70,000 American pupils (9, 13, and 17-year-olds). The students were asked the problem:

"Carol earned d dollars during the week. She spent c dollars for clothes and f dollars for food. Write an expression using d, c and f that shows the number of dollars she has left." (Carpenter et al, 1981).

The results she shared on the problem of translating from one symbol system (natural language) to another (algebraic code) shows the difficulty that novice algebra students have when using a new symbol system when they are not yet familiar with its semantic structure. As the research shows that when students were asked to solve the problem:

"Write an equation using the variables s and p to represent the following statement: There are six times as many students as professors at this university. Use s for the number of students and p for the number of professors."

Most of the students answered 6s = p instead of 6p = s. This shows that some children did not appear to realise that the value of an unknown is independent of the letter used. These children seemed to believe that changing the letter implied that the value was also changed. There were some children whose work showed that they established a sort of correspondence between the order in which letters appear in the alphabet and the number system: letters towards the end of the alphabet were assigned a higher value than those nearer to the beginning.

Many students who have never been taught algebra think that the letters are abbreviations for words, such as 'h' for height or for specific numbers. These numbers were the "alphabetical value" of the letter, such as h = 8 because it was the eighth letter of the alphabet. Another interpretation stems from Roman numerals. For example, 10h would be interpreted as "ten less than h" because IV means "one less than five."

Algebra is an abstract subject where the symbols and letters have different abstract meanings. If students are taught abstract ideas without meaning, there will be no understanding and if teachers want students to know what mathematics is as a subject, they must understand it. When students memorise rules for moving symbols around on paper, they may be learning something but they are not learning mathematics.

Nevertheless, researchers have very different opinions on the possibility of overcoming the difficulties through appropriate teaching. It is true that not all difficulties students face on representing or interpreting symbols are due to certain teaching problems but many problems or misconceptions may derive from meaning and roles that the symbols obtain within the algebraic language (NCTM, 2000). For instance, distinction of a root of an equation from a variable of a function causes serious mistakes in the later interpretations of many functional and non-functional problems.

Through proper teaching strategies and the introduction of algebra right from early education, we can overcome the difficulties of teaching and learning this subject. But even then teachers need to remember that this is an abstract subject which cannot be taught in an abstract way.