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Teachers' understanding of word problems

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What is a word problem?

Mathematics involves both language and numbers with symbols. Questions belonging to equations can be expressed in both symbols (algebraic form) and words. For instance, *"There are 133 stones in two piles. If the second pile has 19 more stones than the first pile, how many stones are in each pile?"* is a mathematical problem which has been expressed in words. As far as the definition of a word problem is concerned, it has no single correct definition. The way teachers/learners interpret word problems might differ from teacher to teacher and learner to learner depending on the degree of difficulty. In my opinion, an equation which is expressed in words rather than in an algebraic form and is not clear to the learner/teacher or is difficult to make meaning of is called a 'word problem'. The mathematical problem given above can also be expressed in an algebraic form as, "$x + (x + 19) = 133$" where 'x' represents the unknown number or variable, in this case the number of stones in the first pile.

Word problems do not sound like mathematical problems but provide a context for mathematics, because mathematics and language are closely related. They are both symbol - systems and action - systems. Herscovics and Kieran (1980) argue that word problems are essential to create relevance for algebra but learners may fail to develop meaning for equations. An article on 'Word Problems' written by Decorte and Verschaffle (1991) reveals that "children's errors on word problems are very often remarkably systematic: they result from misconceptions of the problem situation which are due to an insufficient mastery of the semantic scheme underlying the problem" - in other words, a lack of understanding of the situation and context of the problem and its relationship to mathematics.

Why word problems?

As a learner and a teacher I have often noticed that it is easier to understand an equation when it is expressed in symbols (algebraic form) but it becomes a cognitive obstacle for me to understand when it is expressed in words. What I mean by 'cognitive obstacles' are things that block one's way of making meaning out of a particular problem. In other words, things that hinder the process of acquiring knowledge/ideas in one's mind. I have experienced the following as cognitive obstacles when dealing with word problems in mathematical equations:

1. The ambiguity of the question and use of difficult words

Sometimes an equation expressed in words does not explain exactly what is being asked. In a Secondary Mathematics Textbook I came across the following question: *"Three lots of a number exceeds by 60. Find the number"*. The problem arose when I could not make out the meaning of 'lots' and 'exceeds'. The second confusion was 'exceeds by 60'. I was not sure whether it was exceeding by 60 *from the number itself* or *from the three lots of the number*. How can we expect a child in class 6 or 7 to understand this question and to solve the problem?

2. Lacking language efficiency

Learners, especially those who are not at home (not fluent) in the language in which the equation is expressed, cannot understand the nature of the problem/question and cannot make relationships between known and unknown quantities. They become puzzled in making decisions about which operations to use, e.g. what to add, where to add, what to subtract from what, and what should be on the right-hand side and left-hand side of the equation.

3. Lack of concentration on key words

Sometimes learners do not concentrate on words like *from, of, by, to* etc. Such words can change the nature of the situation given in the equation and learners might come up with the opposite meaning of what has been asked in the question. During my M.Ed. course, I faced the same problem in solving one of the questions given by my tutor. The question was *"I am thinking of a number. If I add 12 and take the result from 100 I will end up with 68. What is the number?"* I read the question and changed it into a symbolic expression, i.e. "$x + 12 - 100 = 68$". After solving the question I checked the answer but it did not match the question. One of my colleagues who was also solving the same question at the same time asked me to share my answer with him. He looked at what I had done and asked me to read the question carefully and then change the description into an algebraic form. After I went back to the question and concentrated on the words, I found my mistake, i.e. I was subtracting 100 from 12 rather than subtracting $(x + 12)$ from 100. I resymbolised the equation as $\{100 - (x + 12) = 68\}$ and found the correct answer. How deceiving this single word "from" was.

This confusion opened up a discussion on such problems in our group. Our tutor also joined the discussion in order to get an insight into such cognitive obstacles. Although it gave only a slight insight into how word problems can be resolved, it was very valuable for me - it opened my eyes to *see* what is written in the question and opened my mind to *reflect* on what the question asks and what I am actually doing. The fear of frustration and failure I felt was replaced by a new courage and zeal for further learning.

Teachers' roles and their teaching styles play a vital part in constructing the learners' relational understanding of word problems. Working on word problems with my two other colleagues during the 'Mathematics Specialisation Module' of the M.Ed. course, we came across a question *"6 years ago a father's age was 6 times that of his son. But after 8 years it will remain only twice!! Find their present ages."* We decided, first, to read the question and to change it into an algebraic expression and then solve the equation but neither of us could make any meaning out of the question. The way we symbolised the equation did not work. We were confused when thinking of different ways to solve this problem. When the tutor (course leader) came to us and wanted to know what we had been doing I gently pushed my notebook and pen towards her, expecting her to solve the problem and to explain where we were stuck. She just started asking questions, such as, "what have you done so far?", "what are the things you have not understood?", what other ways could you try?" and "have you looked at the relationship between so and so?" She did not say even a single word leading to the solution of the problem. She kept on probing each of us till we arrived at the understanding of the meaning of the problem and could symbolise the equation by drawing the following diagram.

TIME	SON'S AGE	FATHER'S AGE
A. 6 years ago	x - 6	6 (x - 6)
B. After 8 years	x + 8	2 (x + 8)
Period from 'A' to 'B' = 14		
'x' represents the number not known - i.e. the son's age *now.*		

Equation = $6(x-6)+14=2(x+8)$

We not only solved the problem but also figured out where the confusion had arisen. Her probing into our minds, as Backhouse *et al.* (1992) argue, stimulated our thinking. We became more conscious in reflecting on what we were doing, and the solution of the problem came out of our own minds rather than the teacher telling us the answer.

What I learned from this experience is that we can enable our students to try to make meaning out of the question (word problem) before looking for the solution. Once they understand what the question asks, what is known and what is not known, then it will be less difficult for them to arrive at the solution. We can also allow them to keep on going through the question (walking into the problem) till they can make a connection between what is known and what is not known. We can encourage them to open up a discussion through which they can get an insight into each others' understanding of the problem.

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