Exploring students' learning difficulties in secondary mathematics classroom in Gilgit-Baltistan and teachers' effort to help students overcome these difficulties

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Exploring Students' Learning Difficulties in Secondary Mathematics Classroom in Gilgit-Baltistan and Teachers' Effort to Help Students Overcome These Difficulties

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Abstract

This article sets out to describe and explain how four high school teachers, identified as improvement-oriented teachers (IOTs), in their day-to-day teaching, try to use pedagogical remedies to help their students overcome the difficulties that hinder in-depth learning in secondary mathematics classrooms. Providing reflective accounts from the IOTs' experiences and presenting illustrative examples from their classrooms, the study provides a broad picture of the context in which students learn mathematics. The study recognizes the factors that constrain students from gaining in-depth understanding into subject matter knowledge; it highlights the possibilities of fostering in-depth learning by establishing the primacy of the teacher in bridging the gap between students' perceptive faculty of mind and subject matter knowledge. It recognizes the influences the teacher's actions, pedagogical moves and decisions exert on students' in-depth learning of concepts. The study also underscores the vital importance of students' prior knowledge of basic mathematical concepts in in-depth learning of new concepts. Implication of the results of the study underscores the need for synergy of efforts on the part of teacher, school, and other key stakeholders, and curriculum in creating and promoting an environment conducive to students' in-depth learning in mathematics.

Key words: National Curriculum, secondary mathematics, mathematical concepts, in-depth student leaning, learning difficulties, pedagogical remedies

Background of the Study

Mathematics, as one of the core curriculum subjects, is taught in all public and private schools in Pakistan from Grad 1 to 10. In the mathematics curriculum prescribed for middle and secondary classes a wide spectrum of concepts are to be learnt and mastered by the students. Generally, learning mathematics is not fun for a majority of students studying in public and private schools in Pakistan but a nightmare. Mathematics curriculum contains specialized knowledge which needs certain attitudes, frame of mind

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(analytical and logical thinking) and efforts on the part of learner (Ellis, 2011; Government of Pakistan, 2006; Rojan, 2008). Unfortunately, in many government and private schools in Pakistan teachers usually fail to instill and nurture these critical abilities in students. The result is that even after passing Grad 10 majority of students fails to make any connection with the subject (Government of Pakistan, 2009). Teachers try to transmit the knowledge to students that is prescribed in textbook, asses students' learning through getting them to define or apply rules in a prescribed way (Amirali & Halai, 2010; Mohammad, 2002). Mohammad (2002) reports, "Mathematics learning consists mainly of memorization of rules for solution of textbook problems. Students memorize rules…without understanding why they are doing any of it" (p.3).

In Pakistan in general and Gilgit-Baltistan in particular there has been little of no systematic, in-depth research into student learning at secondary level in core curriculum subjects like mathematics. However, some studies conducted at primary level have presented a dismaying picture of the level of students' achievement in core curriculum subjects such as science and mathematics. A wide range of factors are responsible for poor quality of mathematics education in main stream government schools in Pakistan. Notably, the factor such as teachers' poor subject knowledge, teachers lacking in pedagogical competence and teachers' and students' perceptions about mathematical knowledge hinder students from developing mathematical understanding Amirali & Halai, 2010).

Yet another important factor which affects students' learning and virtually the level of achievement in is mathematics curriculum. The Mathematics curricula's standards at all levels primary, middles and secondary) are either incompatible with the mental level of students or school Mathematics curriculum has not been thoroughly contextualized to reflect students' interest, aspiration, and above all, their real life experience (Hiebert & Carpenter, 1992). Understandably, when students do not relate well to the curriculum they fall short of efforts to excel in the subject.

To improve teaching and learning processes in Mathematics classrooms requires a better understanding of the real nature of the common difficulties that hinder conceptual learning, particularly at secondary level, as well as the pedagogical remedies by the teachers, to help students overcome these difficulties. The concern primarily arises from a desire to see students learn more than memorization and recalling factual information provided in the textbook, that is what is happening in many mathematics classrooms in Pakistan, where students are treated as 'parrots' rather than 'active learners' or 'creative thinkers'. Despite its high academic value in-depth student learning in mathematics is not usually the focal point of all the classroom activities and interactions in most government and private schools in Pakistan. There is an obvious paucity of the contextualized understanding of the challenges our students and teachers faced everyday in Mathematics classroom and the possibilities of improving mathematical learning that exist in our schools. Against this backdrop, the study was undertaken to seek to create a context for discussion and reflection on issues of and opportunist for improved mathematics education by situating them in the nexus of teaching and learning in the classroom.

Literature Review

Relevant applicable literature was reviewed to know about the existing state of the knowledge on the topic, find theoretical grounding for the study, and to inform the methodological considerations.

The new National Curriculum of mathematics is based on three broad categories of activities that define the critical abilities of scientifically literate students in Pakistan. These are: knowing and using mathematical knowledge (learning science; constructing new science knowledge (doing mathematics; and reflecting on mathematical knowledge (thinking mathematically). These broad performance indicators are connected with standards and benchmarks which describe what knowledge and skills students should acquire in the subject. These standards emphasize "high order thing", "deep knowledge", "substantive knowledge", and "connection to the world beyond the grade room" (Government of Pakistan, 2006).

The ultimate learning outcomes intended in the National Curriculum are quite ambitious, requiring teachers to adopt a comprehensive vision of pedagogies and engage in practices premised on constructivist philosophy of learning which emphasizes centrality of students in the learning process (Fosnot, 2005). How this happens in reality? Overview of the available literature alludes to a wide gulf that exists between the ideal curricular goals and what actually happens in most of mathematics classrooms in Pakistan in general and in rural Gilgit-Baltistan in particular.

A good number of studies with national, provincial and district coverage in Pakistan have been undertaken to assess students' level of achievement in core curriculum subjects including mathematics. These studies have been consistent in reporting students' (Grade 3-5) low level of achievement in Mathematics, compared with other curriculum subjects (Abdeen & Jone, 2000; Academy of Planning and Management, 1999; Benoliel, O'Gara & Miske, 1999; Government of Pakistan, 1999; Government of Balochistan, 1999; Government of Sindh, 2000; Government of Punjab, 1999; Pervez, 1995; Shah, 1984, UNESCO, 1999). These studies have further showed that students performed better on items measuring rote learning and poorly on items requiring comprehension, problem solving and life skills (Academy of Planning and Management, , 1999; Pervez, 1995; Samo, 2009). These studies attribute a wide rage of factors to the low level of students' achievement in Mathematics. However, teachers' academic and pedagogical competence in and their attitude towards the subject has been described as a common denominator in facilitating or hinder students' in depth learning in Mathematics.

A recent study by Tayyba (2010), investigated the mathematics achievement of middle grade students in Pakistan. Specifically the study attempted to determine whether mathematics achievement varies systematically across students and schools; to what extent the mathematics curriculum and frameworks are implemented in schools account for differences in mathematics achievement. Findings of the study indicated that students were able to pass low-rigor items requiring simple mathematical skills. Moreover, items favoring female students in content domain belonged to knowledge of concepts to recall basic facts, terminologies, numbers, and geometric properties. Items favoring male students in either domain belonged to the problem solving level.

Defining and describing in-depth student learning

In the context of mathematics, 'in-depth learning' and 'rote learning' have variously been defined and explained. For example, according to Jenkins (2010), in-depth learning manifest itself in mathematical thinking which is characterized in terms of how students make of sense of mathematics, the strategies they apply to solve problem, the conceptual representation they create, the argument they make and the conceptual understanding they demonstrate. In his highly influential and widely cited paper, Richard Skemp (1976, p. 23) has presented his views on types of learning: "relational" and "instrumental"; the ideas explained in the context of instrumental and relational learning are relevant to the practice of teaching mathematics in any context now as they were presented 35 years ago. Relational learning is explains both as what to do and why (knowing with reason), where as instrumental learning is described as "rules without reasons".

In-depth learning: A valuable educational goal

It is clear from the above discussion that 'in-depth learning' is used as an antonym of 'rote memorization' of content knowledge (information abut phenomena, rules and principles) for the sake of reproduction when required. International research has established a real value of in-depth learning for students. In the literature numerous advantages have been associated with the goal of in-depth learning of subject matter knowledge. Newton (2002), for example, enumerates the following advantages of in-depth learning: First, in-depth learning can satisfy a number of personal needs of the learner. One of the important needs is the desire to achieve a certain level of satisfaction, which arises from the curiosity to know reasons, facts, justification and causes behind events or principles. So in-depth learning helps to meet these demands towards self-satisfaction. Second, in-depth learning accelerates the processes of mastering the new materials and flexible use of knowledge in other context or situation.

Research provides evidence that children who learn subject matter knowledge with thorough understanding demonstrate an enhanced ability to think flexibly when dealing with novel problems (Newton 2002; Sierpinska, 1994). In-depth learning "confers a certain cognitive autonomy on its owner" (Halfords, 1993, p. 165); and it enables the learner to effectively and independently interact with the world and think for themselves and make reasoned choices (e.g., Johnson-Laird, 1985; Kilbourn, 1992; Pettroski, 1993; Prawat, 1989). In-depth learning in mathematics facilitates further learning; it enables critical abilities such as reasoning and analytical skills, and helps develop learners' creative faculty of mind (Newton, 200; Perkins, 1993). Despite being attached much benefits to in-depth learning more often than not it remains a secondary concern in the classroom. Why this is so? Below, some of the possible reasons behind this negligence towards in-depth learning in the classroom are examined briefly.

In-depth learning avoidance: A common phenomenon

Notwithstanding the above explained good reasons to treat in-depth student learning a valuable educational goal, unfortunately it is and has not been a central concern in every classroom in the world in general and in Pakistan in particular. According to Perkins (1993), "…teaching for understanding is not such an easy enterprise in many educational settings. Nor is it always welcome" (P.02) A survey of the current literature reveals that understanding avoidance is not unique to the context of Pakistan and other developing countries where the quality of education is considered to be poor; it is and has been a matter of concern in the context of developed countries as well, where there is a tendency to emphasize memorization and reproduction of information (e.g., Das & Barunah, 2010; McLaughlin & Talbert, 1993; Wildy & Wallace, 1992).

Factors that facilitate or hinder students' in-depth learning

In-depth learning is often difficult because it entails deep cognitive engagement with the subject matter. This is why students do not voluntarily or spontaneously engage in cognitive activity that fosters in-depth learning. Students bring with them a variety of conceptions, abilities, skills, knowledge, interest, attitudes, beliefs, perceptions, aspirations, expectations, habits, and preferences, which may not be in harmony with the demands of deep engagement with subject matter. The literature describes a variety of factors which bear upon students' learning behaviors and abilities. These factors, however, are rooted in two main sources: external forces or environment such as parent, career aspiration, employment's need, etc. and the intrinsic motivation learners bring to the classroom; students with intrinsic motivation and interest may be more inclined to seek understanding while others want to pass the examination (e.g., Davis, 1994; Hart 1981; Gibbs, 1992).

Moreover, the prior knowledge students bring to the learning situation is considered to be a vital factor in facilitating in-depth learning (e.g., Gollub et al., 1993; Perkins, 1993; Mayhill & Brackley, 2004). Prior knowledge of primary concepts provides a foundation upon which learning of subsequent concepts is based. Evolution of mathematical thinking and mathematical reasoning thus becomes a process which can be stimulated or in one way the other be influenced by the external factors or conditions, which, in many researchers' view, could be controlled, to a great extent, by the teacher (Eve & Tirosh, 2008). Researchers have described the teacher's interaction with learners as the axis on which education quality of learning turns (Lockheed & Verspoor, 1991; Stoll, 1999). As Stoll (1999) argues that teacher's beliefs, perception, behavior, teaching strategies, and subject knowledge are likely to determine the degree to which students make sense of the material presented to them. Both National Curriculum 2006 and Education Policy 2009 stress upon a marked shift in teacher's role from transmitter of information to creator of learning environment in classroom which supports students in developing rational understanding of the mathematical concepts.

Thus, teachers' central role in promoting deeper learning requires them to understand and practice some of the basic principles of the conceptual learning in mathematics. These principles include teaching general knowledge or generic concepts in the subject and helping students in overcoming the difficulties they face while mathematical concepts. Teachers can use a wide variety of activities and techniques such as discussion, stories, songs, role play, visual illustrations, patterns seeking, using examples from real life, use of analogy and explanations, to help build prerequisite knowledge and strengthen connections between what students already know about a concept what they need to know more about it (Joseph & Yoe, 2010; McLaren, 2010).

The views just discussed suggest that the way students learn is essentially influenced by the way teacher teaches. Mediocre teaching may result in poor learning of subject matter knowledge. Encouraging students to think logically and learn more relationally is always challenging for both teachers and the students because the investment (in terms of time, efforts, cognitive engagement) required to fostering in-depth learning is greater than instrumental learning which depends merely on rote memorization. In this context, it is important to examine the various roles a teacher can play in and the pedagogical tactics teachers apply in efforts to promote in-depth learning in mathematics classroom. Recognizing teachers' critical role in in-depth student learning in mathematics classroom thus gives rise to such intriguing questions in mind as how do the teachers recognize the worth of and apply appropriate instructional strategies to promote in-depth learning in mathematics; what are the common challenges students usually face everyday in mathematics classrooms; and what instantaneous pedagogical remedies do teachers use to assist students in overcoming these challenges? This study was undertaken to seek better explanation to these questions with the ultimate purpose to bring about change in the teaching and learning practices in mathematics classrooms.

Research Methodology

An exploratory qualitative case study method was used to investigate the topic in hand. It allowed an in-depth investigation of the teachers' perception of in-depth students learning in mathematics, the context and nature of common difficulties students faced everyday in the mathematics classroom and the instantaneous pedagogical remedies the teachers used to help students overcome these difficulties.

Research Questions

The following questions guided data collection in the study:

1. What do the teachers know about the notion of 'in-depth student learning' in mathematics?
2. What common conceptual difficulties do students face in their mathematics classroom?
3. Why do students face these conceptual difficulties?
4. What pedagogical remedies and tactics do the teachers use to help students overcome these difficulties?

The context and participants of the study

The primary respondents in this study were four secondary school mathematics teachers (3 male, 1 female), two belonged 2 government secondary schools for boys while 2 came from a private (non-for-profit school system) schools for girls, characterized by their relatively good reputation for imparting quality mathematics education at secondary level. To study science course (of which mathematics is an integral part) students from other neighboring elementary or high schools would take admission in the sampled schools. The four participants drawn from these schools were selected as a representative of those surveyed in purposive sampling process and all displayed a high level of commitment towards teaching their subject. Moreover, the selection criteria considered to include mathematics teachers in the sample who had at least 5 years of experience in teaching Mathematics and were interested in improving their teaching methods and modifying and adjusting their instruction for the purpose of improved students' learning in Mathematics. The information that helped to identify the schools and the teachers within these schools was collected through visiting schools and district offices.

To help judge the level of participants' commitment to teaching, qualitative information was gathered from head-teachers, teachers, students, and supervisors, and school records (the private school has annual performance appraisal system). Analysis of this qualitative information helped develop portrayal of each teacher from the sampled schools. The characteristics or achievements such as reputation as being hardworking, relatively better performance of their students in Mathematics in Board examinations, and consistent high grading of their performance in appraisal presented them as improvement-oriented teachers who are concern about their students' learning and success.

As far as the academic qualification of the participants is concerned, two of the participants had obtained a Bachelor degree in science (B.SC), with major in Mathematics, Chemistry and Physics, while other two had studied mathematics at higher secondary (Grad 12) level as a major subject. Both had been teaching mathematics for more then 10 years at secondary and middle level. Two of them had also taken a course on 'teaching of mathematics' during their B.Ed and BS.Ed. Studies. All four had participated in short in-service professional development programmes focusing on teaching of mathematics and general science.

Data source

To generate data in the study, a qualitative case study method was employed, which used in-depth interviews, classroom observations, post-observation discussion, and document analysis, as tools for data collection. The case study method allowed in-depth investigation of the teachers' instructional practices and the beliefs and values underlying them (Merriam 1988). Thus the data upon which this article is based comes from transcripts of in-depth teachers' interviews with participants, classroom observations, post observation discussions, and instructional material used by the teachers (e.g., examination of text book content, activity sheets content, etc.). The in-depth face-to-face interviews sought to examine the participants' experiences of promoting in-depth learning in mathematics classroom and their views about the context and nature of students' difficulties.. The interview questions formulated were open-ended, to facilitate in-depth answer and allow the participants to raise issues and reflect on experiences in dealing with these issues.

Data analysis

In line with the research questions, two major categories were used to process the data: (1) teachers' views about conceptual learning of subject matter and the issues underlying it, with particular focused on mathematical concepts prescribed in the curriculum at secondary level, and (2) the ways in which the teachers recognize the conceptual difficulties facing students in in-depth learning of these concepts, how they go about helping students overcome those difficulties. Thus, as a result of content analysis (Miles & Huberman, 1994), a range of themes emerged relating to teachers' beliefs about in-depth learning in mathematics, the contexts that reveal challenges to conceptual learning, the nature of theses challenges and teachers' efforts to help students overcome them. The emerging themes were compared across the four cases and cross-cutting key themes were identified, findings were formulated and key conclusions were drawn from further analysis and interpretation of these findings.

Findings

Analysis of the data collected through in-depth interviews of teachers and observation of routine lessons, particularly the anecdotal evidences and critical incidences recorded during classroom observations shed light on the common difficulties facing students in mathematics classroom, the possible causes underlying these difficulties and teachers' effort to help students overcome these difficulties. Triangulation of data from three different sources leads to formulation of the following key findings which answer the three main questions posed in the study.

First, analysis of the teachers' perspectives about in-depth learning and their reflection on day-to-day teaching experiences explain the meaning the teachers bring to the notion of 'in-depth learning' in mathematics. The teachers seem to bring rather a broad understanding to the notion of in-depth learning, and this understanding in turn seems to influence the way they mediate between students and subject matter knowledge. They consider mathematical learning as a cognitive process rather than an act aimed at memorization of rules. They underscore the need for exposing students' to stimulating leaning environment in early stages to help evolution of mathematical knowledge.

Second, the anecdotal evidences gleaned through classroom observations shed light on the common conceptual difficulties students, the context and nature of these difficulties. There appear to multiple reasons for why students find it difficult to engage with meaningful learning in Mathematics classrooms. However, a close examination of the anecdotal evidence gleaned through classroom observations and the teachers' reflections on this evidence suggest that students' difficulties in grasping mathematical concepts at secondary level are mainly rooted in the knowledge gap they bring with them. Insufficient knowledge of generic concepts seems to hinder students to make conceptual connections. Thus, due to huge gaps in students understanding of fundamental concepts they are unable to engage in in-depth learning of advanced level content in Grade 9 and 10.

Third, the teachers are familiar with the variety of challenges facing students in in-depth learning of subject matter. They demonstrate awareness about the ways through which to address these challenges or at leas lest mitigate the adverse effect of these challenges on students' learning. They try to assist students to overcome the difficulties in their effort to make a good sense of the subject matter presented to them. They do try out their own unique remedial tactics and instructional strategies in order to engage students in meaningful learning. Finding students in difficult situation during lesson they usually resort to such pedagogical moves using examples, questioning, analogies, cues and probing and prompting; offering alternative explanation; supplying information to fill knowledge gaps; reinforcing key points; and back-tracking. These pedagogical tactics are very much grounded in the teachers' personal experiences as learners and resulting intuitive understandings of ways through which to help students make connections and grasp concepts.

The above findings are further examined with the help of the data collected through in-depth interviews, classroom observations and post observation reflective discussions with the teachers.

The teachers' perspectives about in-depth student learning in mathematics

The teachers bring quite an elaborate understanding to what does it me to learn in-depth in mathematics. To elaborate on their concept of in-depth learning they compare traditional and modern ways of teaching and argue about how transition from conventional mode of instruction to a more student-oriented mode of pedagogy could be adopted in mathematics classroom. One of the participants, for example, reflects: "To move away from traditional way of teaching requires us to bring about little but substantive and sustained changes in our routine work. We can make our teaching activity-based or discovery-oriented by adopting a more student-focused teaching practice". To elaborate on this, he says that the "seeds of mathematical thinking" can be sown in early years (primary grades) in students' minds. At early years when children begin to form concepts teachers need to present mathematical concepts by making them as much practical as possible through the use of activities and concrete materials. Student can only master the process of learning mathematics through concrete experience which can ultimately lead to development of students' mental capacity to meaningfully engage in logical reasoning and thinking at the level of abstraction. Concrete experiences not only can help students relate mathematics to everyday life, but also can enhance their motivation and encourages them to actively participate in the lesson. The teacher goes on to elaborate:

> The activities which we carry out with children in mathematics classroom should be stimulating and interesting. This does not need any special environment or extra resources. If teachers' routine practices reflect some creativity they can easily reshape their classroom environment where students become involved in meaningful learning. This necessitates teachers to put little mental effort into planning their daily lessons around concepts in order to make them a bit interesting and interactive. A classroom environment which is genuinely facilitative can only help trigger students' curiosity in exploring mathematics as independent learners (Excerpt from teacher's interview)**.**

Similarly, drawing on his classroom experiences, another research participant talks of the ways through which students could be helped out in overcoming the difficulties they confront while trying to grasp mathematical concepts. In his view, simplification of mathematical problems needs to be tackled in a creative way with students. "Understanding of each concept [in mathematics] involves a few opportunities [either to understand or misunderstand]. If you capture these opportunities you are able to master the process leading to deeper understanding of the concept", the teacher explains. What the teacher has learnt form his experiences is that in case students miss any of critical moment or opportunity to grasp concepts they are in trouble; the opportunities then turn into obstacles in the way of understanding subject matter knowledge. It then becomes the utmost responsibility of the teacher to help student recognize and grasp the crucial steps involved in the simplification of mathematical problems.

Yet another participant, reflecting on one of her recent mathematics lessons, reports: "I taught a mathematics lesson about the algebraic formulae **"$(a+b)^2$"** using blocks. In the first day quite a few students did not follow what I presented; because, for most of the students getting introduced to the concept through a practical activity was an ever first experience, as these students cam from different feeding schools. However, later they understood it very well since we used real material to construct the formula. "Acting as a guide and facilitator, I prepare my students to overcome the problems they are faced with while trying to simplify mathematical problems", the teacher explains (Excerpt form teacher interview).

The above reflective accounts embedded in the teachers' classroom experiences provide insights into their beliefs about and the ways of managing pedagogy geared towards in-depth learning in mathematics. A careful analysis of these accounts reveals important insights, ideas and experiences which are relevant to and helpful in understanding teaching for concepts with deeper learning.

Students' conceptual difficulty and teachers' pedagogical remedies and tactics

There appears to be a huge gape between what is intended in the National Curriculum and what actually happens in the classroom where students learn Mathematics. Realization of such curricular goals as development of higher order thinking, knowing and using mathematical knowledge and constructing mew mathematical knowledge (Government of Pakistan, 2006) remains a utopian dream in the schools. The following Table 1 provides an illustrated picture of the situations that work as barriers to students' in-depth learning, how teachers try to help students overcome these barriers.

As reflected in the above data, almost in all above cited cases students were not able to obtain correct solution to mathematical problem or questions posed by the teacher. The nature and the intensity of individual students' difficulties differ in certain cases. Students' inability to work through mathematical problems or correctly answer series of probing questions posed by the teacher could be attributed to many varied reasons. However, analysis of the above data shows that the common difficulties experienced by majority of students in secondary mathematics classroom are primarily caused by the factors such in-sufficient or imperfect mathematical knowledge of primary mathematical concepts; deeply ingrained conceptual errors; being unaware of the procedures required to correctly perform various operations involved in solving mathematical problem posed by the teachers or prescribed in the textbooks; and lacking in understanding algebraic manipulations (equations, mathematical operations) or adapting their mathematical knowledge to new situations. Thus, students seem to have difficulties in using correct processes and procedures while solving mathematical problems. This may be because of students' lacking command over primary mathematical concepts they are expected to master at earlier stages of their schooling. This concurs with results reported by other studies (e.g., Graybed, 2010; Suurkamm & Vezian, 2010; Yoe, 2010).

Table 1
Examples from participants' classrooms illustrating the context and nature of the common difficulties that hinder students' in-depth learning in mathematics.

S. No.	Concept	Conceptual difficulty hindering in-depth learning	Teacher's action to help students overcome the conceptual difficulty
I	Simplifying Algebraic equations	Teachers: To simplify the equations e.g., $x^2 = 16$, why we should take square root on both sides of the equation? Students (answering in chorus): It is a rule.	By using the analogy of pairs of balance the teacher explained the reasons behind taking square root on both sides of the equation in order to simplify it.
II	Application of algebraic formula	Teacher: Asked students to simply the expression: $\frac{\sqrt{a^2 -1} - \sqrt{a^2 + 1}}{\sqrt{a^2 -1} + \sqrt{a^2 + 1}}$ Students: Could not simply the problem in the first attempt; seemed unable to relate or apply the formula to simplify the given algebraic expressions.	Involving students the teachers simplified first step (multiplying and dividing the expression by $\sqrt{a^2 -1} - \sqrt{a^2 + 1}$ and $\sqrt{a^2 -1} + \sqrt{a^2 + 1}$ respectively and pushed students by giving clue and recalling rules.
III	Why $-x = -3$ Takes the form of $x = 3$	Teacher: Why the minus sign on both sides of the equation disappeared? Students: Remained silent, after a while, one student responded: "It is a rule, Sir".	The teacher explained (verbally) as why minus signs on right-hand side and left-hand side of the equation get cancelled.
IV	Real, Rational and Irrational Numbers	Teacher: How many numbers are there between 0 and 1? Students: Answered in chorus: "No number, Sir" (teacher's question was a bit ambiguous).	The teacher explained that there were uncountable numbers belonging to the Set of Real Numbers and Irrational Numbers, Imaginary Numbers, Fractional Numbers between 0 and 1.

Discussion

By and large, the teachers display awareness about the demerits of rote learning and how it compares with in-depth learning. Retrospectively, the teachers find value in deeper understanding of mathematical concepts. They are cognizant of some of the ways through which conceptual learning can be fostered at secondary level. The emphasis, however, is on quality intervention at primary level into creating mental connection and 'sowing the seed of mathematical thinking' and nurturing it through provision of stimulating environment. The insights reflected from the teachers' description of their practices are helpful in knowing more about the ways and the means through which in-depth learning of mathematical concepts at secondary level could be made easier for students. For example, the teacher's emphasis on mastering some of the critical steps involved in the simplification of mathematical problems makes a lot of sense for those who teach or learn mathematics in difficult situations. Simplification of a mathematical problems may consists of multiple steps, it may involve a few steps (opportunities/obstacles) that are crucial for understanding the whole process of working around the intended answer. Teachers need to pay attention to these vital steps by calling for students' attention, by reinforcing, by asking question, by challenging students, encouraging students to ask questions, by repeating instruction, and by using other strategies such as use of analogy, etc. Teacher should focus these critical steps which if are misunderstood by students can be major hurdle to carrying out simplification. Teacher can maximize his/her gains in term of students understanding of the concept provided he/she knows the ways to emphasize and reinforce these steps.

Reflecting on and analyzing through the above examples we can understand the gravity of the difficulties students face in the classroom. It appears that in the context of each situation presented above the crux of the matter lied in the poor background knowledge students brought to the learning situation. It is evident from the above examples that students of Grade 9 and 10 (age level 13-17) failed to demonstrate rudimentary understanding of very basic but important mathematical concepts contributing to learning of the concepts at higher level. Fundamentally, the challenges seem to arise from students' poor subject knowledge background, which apparently is the consequence of the poor teaching, inadequate academic support and guidance, insufficient individual attention they received from their environment (subject teacher, school, parents and peers). The teachers were of the view that for their poor content knowledge background students might not be blamed, since a substantial majority of students received their primary and middle level education from other public and private school, which did not offer good learning environment. Secondly, the students of their school were taught by different teachers in the primary and middle grades who themselves did not have a command over subject matter knowledge in mathematics.

Further analysis of situations helps to understanding the complexity of the difficulties facing students in mathematics classroom. Below, I examine each of the above examples in turn in order to help us understand as the complexity inherent in in-depth learning of mathematical concepts and how this complexity interacts with the dynamic of teaching and other classroom processes.

Example I: In the above first example **($x^2 = 16$),** the difficult lies in the simplification of the given algebraic equation which needed understanding of other related primary concepts such as square root and power, set, integers and whole number etc. This required the teacher to do back-tracking. He made students to recall the situation in which first time they had been introduced to the concept of square root and power. Students knew how square root is represented symbolically (√), but did not know its meaning or value in mathematical term (e.g., $\sqrt{\mathbf{a}} = \mathbf{a}^{1/2}$) or the basic information in the background of the concept of square root.

Example II: The difficulty students are faced with in recalling and applying or flexibly using algebraic formulae in new and varying situations seems to be a common phenomenon in the mathematics classroom. In the above second example, primarily two problems seem to have contributed to the students' inability to simplify the given algebraic expression. In the given example, the issue at question was finding value (s) for the variable x (unknown), for which purpose students were required to carry out systematic simplification of the given algebraic expression. How to go about this? Where to begin with? The first step in this regard obviously is setting a direction, which can be done by knowing conceptually as what lies at the heart of the question at hand. In the situation at hand, to begin with the simplification process, students needed to manipulate the exiting situation (i.e., multiplying and dividing the expression with such that the given expression gets converted into a generalized formula or rule). In the first instance, students failed to make this connection, which was essentially needed to specify the direction for the simplification.

Given the circumstances, the teacher's intervention was needed. To avoid spoon feeding, the teacher did not write the exact formula but gave a hint (denoting variables by shapes like square, triangle), which was indeed a thoughtful action on the part of the teacher. It, on the one hand, helped break the inertia, produce a little cognitive momentum by making students think and relate their mental structure of the concept to the teacher's example, on the other. As simplification process progressed, a number of conceptual difficulties became evident. Students had problems with minor but important basic concepts such as simplifying square root, manipulating signs (understanding the meaning of the symbol of square root **"√" i.e.** $\mathbf{a}^{1/2} = \sqrt{}$ **or** $\sqrt{\mathbf{a}}.\ \sqrt{\mathbf{a}} = \mathbf{a}$**).** Students got puzzled confronting a situation in which they were required to simplify the simple expression $\{-(a^2 - 1)\}$. In fact, the minus sign out side small bracket seemed to have caused confusion for them. Upon opening the bracket students were required to determine the sign with **1.** From the teacher's reflection it appeared that students did not know for sure that "in multiplication two minus make a plus" leading to the simplification $\{-a^2 + 1\}$ **or** $1 - a^2$**.** The teachers thought that changing the place of **1** and a^2 though was minor step, yet in simplification it is a critically important step because otherwise the expression would not have taken the form $(1)^2 - (a)^2$, which was needed for further simplification i.e. $(1)^2 - (a)^2 = (1 + a)(1-a)$ of the expression.

Examples III: In the above Table 1, the third example also deals with manipulation of signs but the main focus was slightly different. The teacher wanted to assess students' understanding of the reasons that explain as why minus signs with the term on the right-hand-side and left-hand-side of the equation **(-x = -3)** get cancelled. Students' answer suggests that they knew it as an abstract rule; rather than a phenomenon that can physically be explained or understood through examples. There are obvious reasons that underpin generalized formulae and rules extensively used in mathematical calculations and simplifications. Students' deeper understanding of these reasons is highly desirable. This may give students with a great deal of freedom to manipulate the given situation in a variety of ways. For instance, in the example under consideration, students can manipulate the situation in multiple ways to ascertain the reason behind as why the minus signs on right-hand-side and left-hand-side of the equation get cancelled e.g. **- x = -3 or - x + 3 = 0 or 3-x = 0 or 3 = x or x = 3** (since **a = b** has the same meaning as **b = a**). The teacher's example, in which he used an analogy of pairs of scales, was quite helpful in overcoming the conceptual difficulty students were faced with. The teacher explained that if same weight is added in or subtracted from each pan, the balance is preserved, and thus a weight can be found which exactly balances the unknown weight. This justifies taking a number to the other side and changing the sign', since we get the same result from adding certain mass say, 2 Kg. to the left-hand pan, or taking it away from the right-hand pan.

Example VI: This example deals with 'number system in mathematics'. The teacher posed the question: "How many numbers are there between '**0**' and '**1**', raise your hands?" Immediately many hands went up, teacher chose one student to share her answer. "No sir, there is no number between '**0**' and '**1**', the student says". "Is there no number between 0 and 1"?, the teacher asked a follow up question.. "No number, sir", some students replied in chorus. The teacher disagrees with the students saying that there are uncountable numbers between '**0**' and '**1**' including all irrational and fractional numbers.

There seems to be a problem with both the teacher's question and the student's response. The teacher did recognize this. The question was a bit ambiguous as the teacher used the word "number", which means any 'number' within the 'number system' in mathematics. Instead the teacher would have referred to 'irrational' and 'fractional' numbers. Sometimes ambiguous questions further complicate the existing misconception students bring to the concept. Choosing the answer student might have considered the whole number system **(0, 1, 2, 3,…)**.

Conclusion and Implications

The anecdotal evidences discussed above exemplify the typical challenges or the kind of conceptual difficulties students usually face in secondary mathematics classrooms in Pakistan. Not a single factor can be held responsible for students' lacking the cognitive abilities or motivation required to engage in-depth learning; underlying these difficulties there are multiple reasons. However, some of the main reasons that notably contribute to students' lacking the capacity to engage in in-depth learning include their not being conversant with the ways of learning concepts other than memorization because they might not have been exposed to such experiences before (in the previous grades). Their attitude about knowledge and their approaches towards learning mathematical concepts seem to have been shaped by their previous classroom experiences.

Thus, the teachers' reflections together with the data generated by classroom observations allude to students' limited understanding of fundamental concepts as being the primary factor contributing to students' inability to gain command over subject matter knowledge at secondary level. This situation was further compounded by students' hesitation to ask questions when got stuck. Therefore, it is prudent to recognize that the problems that apparently constrain student in-depth understanding of subject matter knowledge in mathematics at secondary level is embedded mainly in the poor prior knowledge background students bring with them. Lacking adequate prior knowledge of concepts is a chronic deficiency, which may not be addressed through easy ways or quick fixes. Both teacher and students need to work hard during the critical stages of students learning primary concepts. Unless schools, specifically teachers pay greater attention to students' in-depth learning of mathematical concepts at primary levels the concern for in-depth learning at senior level would likely to remain a utopian dream. This finding accords with the lessons reported by other studies that emphasize the inherent link between students' prior knowledge and new knowledge (e.g., Gollub et al., 1993; Perkins, 1993; Mayhill & Brackley, 2004).

In sum, in-depth student learning is a worthwhile educational goal. At the same time it is more complex then is being perceived—a tip of ice berg much of it is not visible—and makes teaching harder because of the high demands it places on time, resources, energy, expertise, commitment, and creative mental efforts on the part of both the teacher and the learners. Dealing with the problems on incidental or contingency basis or in a 'crisis management fashion' does not help to overcome the challenges to promoting in-depth student learning in mathematics. Well-thought-through pedagogical decisions and instructional strategies need to aim at constructing conceptual structures and facilitating the process of evolution of mathematical thinking. Teaching for in-depth learning of subject-matter in mathematics therefore needs to be dealt with through careful planning (of content, strategies, time and resources), creative actions and diligent and wholehearted efforts on the part of both the teacher and students.

The above conclusions seem to have important implications for understanding of the realities that exist in secondary mathematics classrooms in Pakistan. Specifically, the implications of the results of this study can be seen and explained in the context of schools, teachers, other education stakeholders, policy and curriculum, which are briefly explained below.

For schools

Students' learning of subject matter with deeper understanding may not take place in the classroom in an isolated fashion. In-depth learning, as explained in this paper, is closely connected with various conditions inside and outside the classroom. Therefore, reforming practices in mathematics classroom calls for synchronization and integration of efforts on the part of school. Synergy can be built around the efforts such as providing opportunities for teachers to enhance their content knowledge, deepen and widen their knowledge of innovative pedagogies and ongoing assessment techniques. This would require schools to recognize the vital importance of long-term planning, preparation and well structured and well-thought through strategies instead of depending on incidental measures to deal with the difficulties arising from teachers' inability to promote in-depth student learning in such important curriculum subjects as mathematics. A majority of secondary schools in Pakistan are composite enrolling students from Nursery to Grade 10. These schools need to pay particular attention to how students learn mathematics in early years. When students move to upper grades with adequate knowledge base and enhanced cognitive skills they can easily master concepts at secondary level. In addition to this, schools need to consider giving a greater degree of freedom for teachers to take certain decisions with regards to syllabus coverage and preparing students for internal and external test/exams.

For teachers

As far as teacher's role in promoting student in-depth learning is concerned, first of all, it is highly relevant to consider as what teachers need to know and be able to do in order to promote deeper understanding of the subject-matter knowledge in mathematics. This inevitably places demand on teachers' knowledge of subject matter, pedagogical content knowledge, knowledge of the learner, knowledge of the curriculum, knowledge of test and evaluation, better understanding of new classroom management strategies, knowledge of resource management, and readiness to accept and ability to cope with the diversified challenges associated with in-depth student learning. The high demands of conceptual learning require mathematics teachers to letting go of transmission-oriented practices; they need to carefully prepare lesson plans, student worksheets, blackboard work, home assignments, and assessment tasks, in order to be able to think about and convey the subject matter in different ways.

For teacher education

Teacher education whether it is pre-or in-service training is considered to be a means to preparation of teacher for the profession. Student in-depth learning would require a marked paradigm shift from teacher-dominated transmission mode of instructions to learner and learning friendly pedagogies. This calls for the need for quality teachers learning and continued professional development. The new or innovative teaching techniques or instructional approaches mathematics teachers in Pakistan need to adopt are to be informed by the knowledge generated by the educational researcher in Pakistan and outside it. It is therefore appropriate to suggest that teacher learning and development in pre-service and in-service programmes may be organized around the knowledge that originates from empirical classroom-based research, evaluation of best practices, studies of successful classroom innovations and life history studies of successful mathematics teachers conducted in both Pakistan and outside it.

For curriculum

Curriculum plays an important role in how students learn and develop in school. The realties of practice, however, suggest that for Mathematics teachers fostering in-depth learning in line with and in the true spirit of the aims and objectives of the National Curriculum is a task easier said than done. It is therefore important to suggest that the goal of in-depth student learning be integrated with the principles that guide school education (focused on secondary level) in general and teacher education (focus on high school teachers) in particular. The goal of fostering learning of subject-matter knowledge with deeper understanding in such core curriculum subjects as mathematics needs to be the first and foremost guiding principles of the school curriculum. If it is not so then the trend or the tradition in which "coverage of syllabus" is considered as synonymous with learning concept would continue to prevail. The syllabus coverage has become almost an 'enemy' of the teachers who want to teach for understanding rather than examination.

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